# Experimental Investigation of the Influence of Stacking Sequence and Delamination Size on the Natural Frequencies of Delaminated Composite Plate

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**Abstract.** The vibration properties of the composite structures is critical to the reliability and durability of the structures. Vibration becomes worst in case the delamination is present within laminates. In this research work, experimental, finite element and analytical techniques have been applied in order to analyze the influence of stacking sequence and delamination sizes on the natural frequencies of carbon fiber reinforced polymer composite (CFRP) plate with and without delamination. The boundary conditions in this research work was (SSSS) (all sides simply supported. Experiments were performed to study the vibration characteristics of (CFRP) delaminated composite plate. Software package ABQUS was used to model and analyze the vibration response of carbon fiber reinforced polymer composite plate for (SSSS) boundary condition and the effect of stacking sequence and delamination size was calculated. Rayleigh-Ritz Method was used to find the natural frequencies for different delamination sizes and stacking sequences. The results was concluded that natural frequencies were significantly affected by the delamination sizes and stacking sequences of (0/90/45/90) showed higher values of natural frequencies in lower mode subjected to all-sides simply supported boundary conditions. It was interesting to see that there were small differences in values of natural frequencies among the stacking sequences for lower modes but the difference gradually increased in case of higher modes.

Keywords: experimental vibrations, delamination, composite plate, finite element analysis, simulations

## Introduction

Composite materials have many advantages when compared to conventional materials (Karsh et al., 2018; Imran et al., 2018 and b). They are lightweight, cheaper, stiffer, stronger and eco-resistant (Kumar et al., 2017; Bakis et al., 2002). A good example of a composite material is carbon fibre reinforced polymers. To form a carbon fibre reinforced polymer, two components are required. They include reinforcement and a matrix. The reinforcement needed is achieved through the carbon fibre. It is the reinforcement of the carbon fibre that gives the required strength to the composite material being formed. The matrix on the other hand provides bonding between the reinforced materials. and an example of a matrix is epoxy (made from polymer resin) (Imran and Badshah, 2012; Bakis et al., 2002). Usually, the characteristics of the carbon fibre reinforced polymers formed depend on the reinforcement and the particular matrix used. For example, reinforcement components like elasticity and stress determines the rigidity and

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strength of the carbon fibre reinforced polymers. This is why the property strength of carbon fibre reinforced polymers is usually described as directional and distinct from isotropic structures like aluminum. It must not be forgotten that the carbon fibre layout also has the ability to influence the carbon fibre reinforced polymers properties (Corum et al., 2000). Composite laminates are preferred due to their many advantages such as their high-strength, bending stiffness and resistant to expansion (Agarwal et al., 2017; Imran, 2015). Usually, composite laminates are formed from a hybrid of fibre matrix layers. The fibre usually consists of such materials as boron or glass, while the matrix consist of such materials as aluminum or epoxy. Undoubtedly, Fibre reinforced composites have gained wide use in many aspects of engineering applications. They have been applied in ship building, mechanical, aerospace, etc. They are preferred because of the advantages they have such as superior mechanical characteristics, highstrength, their stiffness ratio to weight is excellent (Friedrich and Almajid, 2013). One of the crucial parameter in composite laminates is delamination that are generated accidentally either during service period or during manufacturing of the composites (Saghafi *et al.*, 2017; Yelve *et al.*, 2017; Kharghani and Guedes, 2016).

Structures prior to be applied in service should be investigated (Imran *et al.*, 2019 a,b& c; Shao *et al.*, 2017). Delamination is the sensitive defect that affect the vibration characteristics of carbon fibre reinforced polymer composites (Imran *et al.*, 2018 a&b; Venkate *et al.*, 2017) so, it is quiet important to study the influence of stacking sequences and delamination sizes (Imran *et al.*, 2019 a,b&c).

A lot of studies have been done to investigate the vibration properties of composite structures like composite beam, composite shells, glass fiber reinforced polymer composite structures, however studies to find the effect of different parameters like stacking sequences and delamination sizes are scarce (Imran *et al.*, 2019 a,b&c). In following paragraphs, we will do literature studies to find the effect of the above parameters on the vibration characteristics of different composite structures.

Shu and Della (2004) studied the free vibration impact of composite beams subjected to multiple delaminations. Classical Euler-Bernoulli was used as governing equation. It was concluded that primary and secondary frequencies decreases as the length of the delamination increased, however reduction of primary frequency is not significant. In this study the size effect of delamination was also analyzed. No effect on frequency for short delamination was observed contrary to large delamination that reduced the natural frequency significantly. It was also observed that natural frequency and mode shapes are largely dependent of the assumed constraints. Jadhav and Bhoomkar (2016) carried out experimental and finite element analysis of cracked composite laminated beam.

Lee *et al.* (2002) conducted the vibration analysis and analyzed the effect of multi-delamination on the vibration properties of multi-delaminated composite beam columns. Loading was axial compressive. It was shown that effect of multi delaminations are more significant than those for single delamination. Kim and Hwang (2002) investigated the influence of delamination. They found that the natural frequencies and vibration mode shapes in honeycomb sandwich beams. Delamination was embedded between the face layers of carbon/epoxy laminates and nomex-aramid honeycomb cores. Equation of motion of the split sandwich beam was used for debonded honeycomb sandwich beam. An increase in value of debonding or delamination. They observed that value of natural frequency reduced. At delamination length of 50 mm, a critical debonding point was observed at beyond which response of natural frequencies was unpredictable or disproportional. Also mode shapes become smaller for the increase in debonding size. The

theoretical results obtained in this analysis were close

to the experimental measurements.

Brandinelli and Massabò (2003) conducted analysis to find the free vibration characteristics of beam-type composite structures subjected to delamination with proposed analytical solution using the mid-plane delamination and constrained model assumptions. They neglected the shear deformations and considered it as homogeneous material. It has been observed that opening mode continues to disappear as the transversal longitudinal spring stiffness tends to increase from zero to higher values. Luo et al. (2004) did an investigation to find the non-linear vibration properties of the composite beam subjected to variation in sizes and positions of the delamination. Since the amplitude is small, larger value of frequency would be observed with greater length of delamination. Frequency increases relatively slow with an increase in length of the delamination. As the value of amplitude is increased, the influence of the positions or locations of delamination gets clearer. The influence by transverse shear deformation has significant importance and cannot be considered as no impact for non-linear vibration characteristics of composite beam.

Nanda and Sahu (2012) find the vibration response of delaminated composite shell using first order shear deformation theory subjected to cylindrical, spherical and hyperboloid shells. The value of linear frequency showed increasing trend with reduction of number of layers. This frequency difference continues to increase after the six number of layers. Moreover, higher modes were more influenced by the delamination. Yam *et al.* (2004) analyzed the dynamic behaviour of composite plate with multi-layered structure subjected internal delamination. Eight-node rectangular thin element was used for finite element formulation. It was concluded that local internal delamination had slight or negligible influence on the natural frequencies of laminated

composite plate with multi-layers. Moreover it was also found that natural frequency decreases if the area of delamination is increased. Kim *et al.* (2003) found the effect of delamination and without delamination on cross-ply laminates with various delamination at multiple places. They used layer-wise composite laminate theory. It was observed that natural frequencies decreased with delamination and there is significant difference between delaminated and without delaminated structure.

The vibration characteristics of delaminated composite laminates using finite elements and used third order shear deformation theory which studied by (Thornburgh and Chattopadhyay, 2003; Thornburgh and Chattopadhyay, 2001). Ply thickness was reduced, while incorporating matrix cracking.

Mohammed (2017) carried out vibration analysis of cantilever composite beam produced by hand-lay-up method. They used Matlab, Solidworks and did experiments to find the effect of fibre angles on the mode shapes and natural frequencies. Juhász et al. (2017) developed a model using finite element analysis predicting the modal analysis in through-width delaminated composite plate and the results were validated experimentally. Zhang et al. (2018) conducted finite element and experimental analysis to find the vibration properties of carbon fibre and glass fibre reinforced polymer composite plates. They analyzed the influence of delamination size and delamination location on the natural frequencies. (Tsongas et al., 2017: Vo et al., 2017) carried out free vibration analysis using ANSYS and ABAQUS and compared the results with numerical ones obtained from shear and normal deformation theory on the axial loaded composite beam.

From the above literature, it is observed that the vibration analysis subjected to (SSSS) boundary conditions with different delamination sizes for these three stacking sequences (0/45), (0/90) and (0/90/45/90) is very poor. Therefore, it is utmost important to investigate the vibration characteristics of carbon fiber reinforced polymer composite plate.

Finite element approach using ABAQUS. It shows delaminated region modeled in ABAQUS software package version 16. In order to find the dynamic characteristics of delaminated carbon fibre reinforced composite plate, SOLID (Eight-node brick element) was used as element type and average number of elements per model were 180000. The orientation of the layers were modeled using coordinate system. All delamination surfaces are located in the middle-plane of the composite laminated plate as shown in Fig. 1.

A total of 48 models each consisting in at least 12 natural frequencies and their corresponding modal shapes were computed.

### **Materials and Method**

The composite plate samples were prepared using handy layup method (Imran *et al.*, 2019) and cured at room temperature. Teflon tape having sizes of 6.25%, 255 and 56.25% was used as delamination in the center of the composite plate and layers were stacked as per their configuration like (0/90/45/90), (0/90) and (0/45). In this case, eight layers of laminates were joined together using 50:50 fibre to matrix weight with mid plane delamination. Free vibration analysis was conducted by using Model Hammer Model 2302 and FFT analyzer Model 3560 (Karim11 *et al.*, 2017; Tsongas *et al.*, 2017; Mishra & Sahu, 2012) as shown in Fig. 2. Simple layout



Fig. 1. View of mesh created for composite plate.



Fig. 2. Simple layout of Experimental Setup.

of experimental setup The natural frequencies of all plates subjected to all sides simply supported were determined with and without delaminated plate.

**Analytical analysis.** Rayleigh Ritz Method (Oliveri and Milazzo, 2018; Vescovini *et al.*, 2018; Ardestani *et al.*, 2017; Sayyad and Ghugal, 2017) is used to find the natural frequencies as in equation (1)

where:

A, B, C and D are frequency coefficients dependent on the boundary conditions of the structure. After putting the values of all sides simply supported boundary conditions in the equation (2), we will solve this in Matlab tool to analyze the natural frequencies.

$$\omega_{m,n} = \frac{\pi^2}{a^2 \sqrt{\rho}} \sqrt{D_X m^4 + 2D_{x,y} m^2 n^2 (\frac{a}{b})^4} \cdots (2)$$

where:

a = width of plate; b = height of the plate;  $\rho =$  density of the plate.

# **Results and Discussion**

Natural frequencies values obtained from three techniques, i.e ABAQUS, analytical and experimental methods have been presented in Table 1. Results showed that all the three techniques are in good agreement for results and the error percentage is less than 2% in all cases which validate the methods adapted for all sides simply supported delaminated carbon fibre reinforced polymer composite plate subjected to stacking sequences of (0/90/45/90), (0/45) and (0/90).

 Table 1. Comparison of results from experimental, analytical and finite element methods subjected to all sides simply supported boundary conditions and without delamination

	ABAQUS	444.7	1175.8	1175.8	1769.6
0/90/45/90	Analytical	447.6	1182.3	1182.3	1790.6
	Experimental	425	1168	1179	1777
	ABAQUS	475.9	1191.9	1191.9	1892.9
0/45	Analytical	482.4	1206	1206	1929.6
	Experimental	468	1189	1185	1895
	ABAQUS	421.6	1163.1	1163	1678.2
0/90	Analytical	422.2	1165.8	1165.8	1688.8
	Experimental	409	1151	1160	1674
	0/90/45/90 0/45 0/90	ABAQUS 0/90/45/90 Analytical Experimental ABAQUS 0/45 Analytical Experimental ABAQUS 0/90 Analytical Experimental	ABAQUS444.70/90/45/90Analytical447.6Experimental425ABAQUS475.90/45Analytical482.4Experimental468ABAQUS421.60/90Analytical422.2Experimental409	ABAQUS       444.7       1175.8         0/90/45/90       Analytical       447.6       1182.3         Experimental       425       1168         ABAQUS       475.9       1191.9         0/45       Analytical       482.4       1206         Experimental       468       1189         0/90       Analytical       422.2       1165.8         Experimental       409       1151	ABAQUS         444.7         1175.8         1175.8           0/90/45/90         Analytical         447.6         1182.3         1182.3           Experimental         425         1168         1179           ABAQUS         475.9         1191.9         1191.9           0/45         Analytical         482.4         1206         1206           Experimental         468         1189         1185           0/90         Analytical         422.2         1165.8         1165.8           Experimental         409         1151         1160

**Table 2.** Effect of delamination sizes on the natural frequencies of delaminated composite plate subjected to SSSS boundary condition.

Delamina-	Stacking	M	lode 1	М	ode 2	Μ	lode 3	Мо	ode 4
tion area	sequence	Experi- mental	ABAQUS	Experi- mental	ABAQUS	Experi- mental	ABAQUS	Experi- mental	ABAQUS
6.25%	0/90/45/90	390	444.78	990	1175.7	1185	1175.7	1800	1769.6
25%		387	441	985	1174.9	1169	1174.9	1755	1769.6
56.25%		385	439.18	983	1172.4	1155	1172.4	1740	1769.4
6.25%	0/45	385	421.66	1100	1163	1170	1163	1701	1678.2
25%		384	411.66	1090	1162.1	1154	1162.1	1689	1678.2
56.25%		384	407.65	1080	1159.3	1146	1159.3	1669	1678
6.25%	0/90	379	475.96	1150	1191.7	1179	1191.7	1851	1892.9
25%		373	473.96	1140	1191	1167	1191	1837	1892.8
56.25%		371	470.95	1120	1188.7	1155	1188.7	1829	1892.7

Effect of delamination size. In the following paragraphs, we will study the effect of delamination size of 0%, 6.25%, 25% and 56.25% subjected to (SSSS) boundary condition. The Table 2 reveals that the on increase in delamination reduces the natural frequencies of carbon fibre reinforced composite plate for (0/90/45/90) for (SSSS) boundary conditions. Experimental and finite element results showed a declining trend with increase in delamination sizes. Maximum value of natural frequency is observed in case of (0/90/45/90) stacking sequence subjected to 6.25% delamination size. This value continue to decrease on an increase of delamination size. One can also see that the higher modes have larger difference than lower modes.

It has been observed that all sides simply supported boundary conditions subjected to all delaminated sizes, with stacking sequence of (0/90/45/90) has the highest values of natural frequencies. It may be interesting to see that natural frequencies are less affected in lower modes for all delaminated sizes.

**Effect of stacking sequence.** From Table 3, it is clear that Mode 1 is less affected in all sizes of delamination subjected to all stacking sequences. Mode 2 and Mode 3 have much lower difference in the values of natural frequencies in case of all sides simply supported boundary conditions. Maximum influence on the natural frequencies is observed. It is also observed that there is significant decrease in natural frequency with and without delamination.



**Fig. 3.** Natural frequency of non-delaminated composite plate subjected to (SSSS) boundary condition.



Fig. 4. Natural frequencies for first four Modes subjected to 6.25% delamination.

Delamination area	Stacking sequence	Mode 1 experimental	Mode 2 experimental	Mode 3 experimental	Mode 4 experimental
0%		444.78	1175.8	1175.8	1769.6
6.25%	0/90/45/90	390	990	1185	1800
25%		385	985	1169	1755
56.25%		387	983	1155	1740
0.00%		475.96	1191.9	1191.9	1892.9
6.25%	0/45	385	1100	1170	1701
25%		383	1090	1154	1689
56.25%		384	1080	1146	1669
0.00%		421.66	1163.1	1163.1	1678.2
6.25%	0/90	379	1150	1179	1851
25%		381	1140	1167	1837
56.25%		380	1120	1155	1829

Table 3. Effect of stacking sequence for delaminated and non-delaminated composite plate.

Highest values of natural frequencies are observed in the stacking sequence of 0/45 without delamination subjected to all four Modes as shown in Fig. 3 and it may be noted that stacking sequence of (0/90/45/90)has highest values in Mode 1 and stacking sequence of (0/90) has the highest values of natural frequencies as shown in Fig. 4.

## Conclusion

It has been concluded that SSSS boundary constraint showed highest values of natural frequencies subjected to 0/90/45/90 stacking sequence for the delamination sizes of 0%, 6.25%, 25% and 56.25%. However lower values of natural frequencies were observed in lower modes. The combined analytical, experimental and finite element analyses of the nonlinear properties are believed to enhance the understanding of the vibration behaviour of the carbon fibre reinforced polymer composites. The results of this analysis can be considered as a base point for the safe, reliable design of composite structures with and without delamination. Based on the experimental, analytical and finite element methods, it has been observed that natural frequencies in case of all sides simply supported boundary conditions get inflected not only by the stacking sequences but also by the variation in delamination sizes.

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**Conflict of Interest.** The authors declare no conflict of interest

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