Pak. j. sci. ind. res. Ser. A: phys. sci. 2025 **68A**(1) 1-7

Effect of Energy and Density Separation of Electron Beams on the Peak Growths and Wavenumbers in a Free-electron Laser with Two Relativistic Beams and a Helical Undulator

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(received July 28, 2022; revised April 24, 2024; accepted April 25, 2024)

Abstract. This article is devoted to the study of the free-electron laser (FEL) employing two relativistic electron beams and a helical undulator. Our study is based on the framework of kinetic description. By using the first order principle the dispersion relation (DR) which has shown the resonance bands is obtained. To generality a water bag distribution function (DF) for electron beams is considered. Using the iteration method by numerical solution DR for the electromagnetic mode and space charge mode has been solved. It has shown that increasing the energy separation of two electron currents results in decreasing the peak growth and causes to increase in the wave number of the FEL resonance, while it has the opposite behaviour for two stream FEL resonance. It has been noticed that the coupling of the electromagnetic wave (EMW) with the space charge wave leads to the instabilities.

Keywords: free-electron laser, relativistic electron beams, dispersion relation, peak growth, resonance band, water-bag distribution function

Introduction

There is a great interest in free electron lasers (FELs) which are electro magnetic (EM) radiation sources, that have some features such as high frequency, coherency and tunable. However, aside from the history of this fascinating topic, what matters now is that in all FEL configurations, an electron beam propagates in a sinusoidal or helical trajectory through the interaction region along with a waveguide EMW in one line (z-direction). An essential condition for lasing is phase matching between EMWs and plasma waves. In this condition, energy can be transferred from the electrons to the EMWs (amplification regime) (Saldin *et al*., 2000). The technology of using laser radiation to increase the speed, precision and delicacy of processing in various fields such as industries, medicine, science, and agriculture is not hidden from anyone. Among them, it is possible to treat by FELs which provides more easily recyclable food to package, durability attractive carpeting and more versatile composite materials. Some other applications are an analysis of peptides and oligosaccharides (Fukui *et al*., 2006) as well as vibrational excitation of the *v*2 mode of NH_3 by IR FEL photon dissociation spectroscopic (Ogi *et al*., 2006).

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Achieving laser like radiation in the X-ray region which is about nano to sub-nano meter wavelength is a breakthrough progress. However, there are some practical obstacles to common FELs in the production of an electron beam with the highest energy and the design and fabrication of undulators with the lowest periods. To overcome this shortcoming scientists have introduced some novel schemes. Huge kind of literatures have been done on the two beam FELs to do so, (Aghamir and Mahdizadeh, 2012; Rouhani *et al*., 2009; Mehdian *et al*., 2008; Liu *et al*., 2006; McNeil *et al*., 2004; Kulish *et al*., 2003).

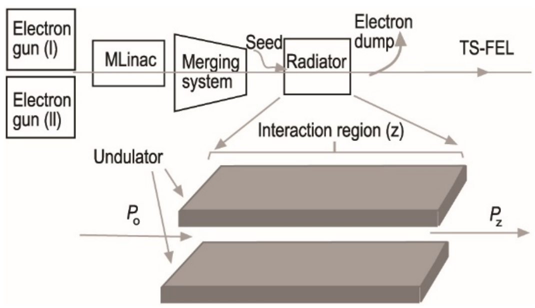

Fig. 1. Schematic setup of a FEL with two REBs (in amplifier mode). Here, P_0 and P_z refer to the initial and final photon radiation power, respectively, in which $P_z>P_0$.

Figure 1 shows a sketch of a two stream FEL system, in which two electron guns and a multi-channel linear induction accelerator (MLINAC) instead of one gun and a LINAC with one channel, in common FELs have been seen. A merging system has been provided to adjust a two-velocity (relativistic electron beam) REB, however, it has shown in an arbitrary scale.

Two stream instability plays a major role in space plasma and dispersion relation is an important relation which well has described considered model. For instance, Dey *et al*. (2022) in the framework of the fluid equations have obtained a dispersion relation and studied particle escaping mechanism from the Ionosphere of Venus (Dey *et al*., 2022). Using the thermal anisotropy 1D-Quantum hydrodynamic model influence of quantum diffraction, relativistic degeneracy and electron spin exchange on the two stream instability in a sample like the Sun have been explored (Sarkar *et al*., 2022; Sarkar *et al*., 2021). The linear wave-particle interaction problem is the main aspect of this paper. However, a similar of this scenario with a nonlinear description as wave-wave interaction has been done for semiconductor junction diode reported by (Goswami *et al*., 2022), as well as in the fundamental plasma (Ghosh *et al*., 2022; Ghosh *et al*., 2021; Mukhopadhyay *et al*., 2021).

Investigation of the thermal effects of the electron beams on the growth rate using the fluid equations for two stream FELs have been done by (Mahdizadeh and Aghamir, 2013). Fundamental properties and some models of FELs with two ERBs with several harmonic have been verified by Kulish *et al.* (2003). The hierarchical nature of the model as well as the nonlinear structure of the harmonic up-conversion of the FELs with two REBs have been studied (Kulish *et al*., 2003). Their work followed by Liu *et al.* (2006) in which they added an axial guiding magnetic field (AGMF) to the system and showed resonance in a two-beam FEL is stronger than common FEL in the same condition at the suitable separation velocity (Liu *et al*., 2006). However, there are not any diagrams or figures to show the suitable separation velocity. This importance has been compensated in this contribution.

One way to find the wavelengths in which resonances should occur and amplification should be expected is the dispersion and gain relation of the considered model. This motivated us reported by (Davidson, 2001) we have derived the DR with rectangular DF. Here, the effects of electron beam energy and density separation on the growth of an FEL with two REMs based on the reportedwork by (Razaghzadeh *et al*., 2023) explored.

The organization of the manuscript is as follows. In section 2, descriptive equations are briefly mentioned. The general dispersion relation (DR) is brought in the third section. In section 4, with the numerical analysis effects of REBs energy and density separation on the scaled peak growths and wavenumbers of FEL and two stream FEL resonances are presented. At the end, conclusion is mentioned.

Descriptive equations. Consider two REBs with unequal velocities $\vec{v}_\alpha$, (the subscript α=1, 2 is an index of the REB) moving through a bifilar magnetic undulator field that is described by

$$\vec{B}(z) = B_w[\cos\theta\,\hat{x} + \sin\theta\,\hat{y}] \quad (1)$$

In equation (1) B_w refers to the undulator field amplitude, $\theta = 2\pi z/\lambda_w$, in which, λ_w is the undulator wavelength, $\hat{x}$ and $\hat{y}$ refer to the unit vectors of the x and y-axis, respectively. For perturbations with small-wavelength, just the effect of longitudinal geometry is considered and the transverse component of thermal effect is neglected. Vlasov-Maxwell Eq. reads (Davidson, 2001; Vlasov, 1938).

$$\left[\frac{\partial}{\partial t} + v_{0\alpha}\frac{\partial}{\partial z} - e\left(\delta\vec{E} + \frac{\vec{B}_w + \delta\vec{B}}{c}\right)\cdot\frac{\partial}{\partial \vec{p}_\alpha}\right] f_\alpha(z, p_\alpha, t) = 0 \quad (2)$$

Here, $\vec{B}_w + \delta\vec{B}$ refers to the perturbed magnetic field, $\delta\vec{B} = \vec{\nabla}\times\delta\vec{A}$, $\delta\vec{A} = \delta A_x(x,t)\hat{e}_x + \delta A_y(x,t)\hat{e}_y$ is the transverse perturbation vector potential and $\delta\vec{E} = -\vec{\nabla}\varphi - \frac{1}{c}\frac{\partial\delta\vec{A}}{\partial t}$. The DF is given by (Davidson, 2001).

$$f_\alpha(z,\ p_\alpha, t) = \hat{n}_\alpha\delta\left(p_{\alpha x} - \frac{eA_x(z,t)}{c}\right)\delta\left(p_{ay} - \frac{eA_y\,(z,t)}{c}\right)F_\alpha(z, p_{\alpha z}, t) \quad (3)$$

Here, $A_x(z,t)$ and $A_y(z,t)$ are the transverse components of the total vector potential. The scaled DF F_α is given by $\hat{n}_\alpha\int dp_{z\alpha}F_\alpha(z, p_{\alpha z}, t) = n_\alpha(z,t)$.

Where, $n_\alpha(z,t)$ and $\hat{n}_\alpha$ are variable and constant average densities of the REBs, respectively. The suggested solution of the nonlinear Vlasov-Maxwell equation is the DF $F_\alpha(z,p_{\alpha z},t)$ (Davidson, 2001; Vlasov, 1938).

$$\left[\frac{\partial}{\partial t} + v_{\alpha z}\frac{\partial}{\partial z} - \frac{\partial}{\partial z}\hat{H}_\alpha(z,p_\alpha,t).\frac{\partial}{\partial P_{\alpha z}}\right] F_\alpha(z,p_{\alpha z},t) = 0 \quad (4)$$

Here, $\hat{H}_\alpha(z,\ p_\alpha,t) = \gamma_{T\alpha}(z,p_{\alpha z},t)mc^2 - e\delta\varphi(z,t)$, the quantity $\delta\varphi(z,t)$ refers to the electrostatic potential and the total electron energy read.

$$\gamma_{T\alpha}(z,p_{\alpha z},t) = [1 + \frac{p_{\alpha z}^2}{m^2c^2} + \frac{e^2}{m^2c^4}(A_{0xw}(z) + \delta A_x(z,t))^2 + (A_{0yw}(z) + \delta A_y(z,t))^2]^{1/2} \quad (5)$$

Using the Coulomb gauge as a condition in Maxwell's equations one can be written (Razaghzadeh *et al.*, 2023; Davidson, 2001; Vlasov, 1938).

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\delta A_\perp(z,t) = -\sum_{\alpha=1}^{2}\frac{\omega_{b\alpha}^2}{c^2}[(A_{0\perp w}(z) + \delta A_\perp(z,t))\int\frac{dp_{z\alpha}}{\gamma_{T\alpha}}F_\alpha - A_{0\perp w}(z)\int\frac{dp_{z\alpha}}{\gamma_{T\alpha}}F_{0\alpha}] \quad (6a)$$

$$\frac{\partial^2}{\partial z^2}\delta\varphi = 4\pi e\sum_{\alpha=1}^{2}\int\hat{n}_\alpha dp_{z\alpha}(F_\alpha - F_{0\alpha}) \quad (6b)$$

Here, $F_{0\alpha}(p_{z\alpha})$, $F_\alpha(p_{z\alpha})$ and $\omega_{b\alpha}^2 = \frac{4\pi\hat{n}_\alpha e^2}{m\gamma_\alpha}$ are the equilibrium DF, the solution of the nonlinear equation (4) and the plasma frequency, respectively. The exact nonlinear evolution of the system is described by the equation (4) along with equations (6a) and (6b) for small perturbations about the beam equilibrium. Using first-order principle at equations (4), (6a) and (6b) in the condition of the small amplitude perturbations about the equilibrium state, one could derive the equation.

$$c^2k^2D_L(k,\omega)D_T(k-k_w,\omega)D_T(k+k_w,\omega) = \frac{1}{2}a_w^2[D_T(k-k_w,\omega) + D_T(k+k_w,\omega)]\{\chi_1^2 - c^2k^2D_L(k,\omega)(\chi_2(k,\omega) + \alpha_3)\} \quad (7)$$

Here, $a_w = eB_w/mc^2k_w$ refers to the scaled cyclotron frequency. A detailed derivation of the DR can be found by (Razaghzadeh *et al.*, 2023). In Eq. (7) the susceptibility coefficient, the transverse and the longitudinal dielectric functions.

$$\chi_n(k,\omega) = mc^2\sum_{\alpha=1}^{2}\omega_{b\alpha}^2 k\int\frac{dp_{\alpha z}}{\gamma_\alpha^n}\frac{\partial F_{.\alpha}/\partial p_\alpha}{\omega - kv_{0z\alpha}} \quad (8a)$$

$$D_T(k\pm k_w,\omega) = \omega^2 - c^2(k\pm k_w)^2 - \sum_{\alpha=1}^{2}\omega_{b\alpha}^2\int\frac{dp_{\alpha z}}{\gamma_\alpha}F_{.\alpha}(z,p_{\alpha z},t) \quad (8b)$$

$$D_L(k,\omega) = \frac{m}{k}\sum_{\alpha=1}^{2}\omega_{b\alpha}^2 k\int dp_{\alpha z}\frac{\partial F_{.\alpha}/\partial p_{\alpha z}}{\omega - kv_{0z\alpha}} + 1 \quad (8c)$$

are presented, respectively. Equation (7) is a general DR for all coupled made, such as space charge, the right and the left circularly polarized EMWs. This DR can be used to determine the resonance band and its growth of a FEL with two REBs. For the case in which the undulator is relativity weak, the resonance plasma wave $D_L(k,\omega) \approx 0$ couples to the resonant right circular wave $D_T(k-k_w,\omega) \approx 0$, which results the circular wave resonant, i.e. $D_T(k+k_w,\omega) \neq 0$. If one neglects the terms containing a_w^2 then obtains.

$$c^2k^2D_L(k,\omega)D_T(k-k_w,\omega) = \frac{1}{2}a_w^2\chi_1^2 \quad (9)$$

General dispersion relation. The longitudinal thermal effects are considered by a rectangular DF for REBs that read (Babaei and Maraghechi, 2008; Vlasov, 1938).

$$F_{.\alpha}(p_{\alpha z}) = \begin{cases}\frac{1}{2\delta_{b\alpha}}, & |p_{\alpha z} - p_{0\alpha z}| \le \delta_{b\alpha} \\ 0, & |p_{\alpha z} - p_{0\alpha z}| > \delta_{b\alpha}\end{cases} \quad (10)$$

Here, $p_{0\alpha z}$ is the average of z component of momentum and $\delta_{b\alpha}$ is a measure of the momentum deviation, which is related to the thermal motion of the REBs.

The condition $\delta_{b\alpha} \ll 1$ is considered to calculate of the integrals in Eqs. (8a-8c). So, $\gamma_{0\alpha}$ and $v_{0i\alpha}$ which appear in the integrals of equations (8a-8c) is expanded to the first order in $p_{\alpha z} - p_{0\alpha}$. We obtain $\gamma_\alpha^{-n} = \gamma_{0\alpha}^{-n} - n\gamma_{0\alpha}^{-n-1}\beta_\alpha(p_{\alpha z} - p_{0\alpha})/mc$. Where, $\beta_{\alpha z} = v_{\alpha z}/c$, $\beta_\alpha = p_{0\alpha}/mc\gamma_\alpha$ and $\gamma_\alpha = (1 -$

$\beta_\alpha^2)^{-1/2}$, $\gamma_\alpha^{-2} = \gamma_{0\alpha}^{-2} + (a_w/\gamma_{0\alpha})^2$. Inserting the rectangular DF, Eq. (10) into the equations (8a-8c), one obtain $D_L(k,\omega)$, $D_T(k \pm k_w, \omega)$ and $\chi_n(k,\omega)$. Subsequently, inserting them in Eq. (7) result DR of the FEL with two warm REBs (Razaghzadeh *et al.*, 2023).

$$c^2k^2D_L(k,\omega)\,D_T(k - k_w,\omega)D_T(k + k_w,\omega) = \frac{a_w^2}{2}\left[\chi_1^2 - c^2k^2(\chi_2(k,\omega) + \alpha_3)D_L\right] \qquad (11)$$

Numerical analysis. By the numerical analysis of the general DR of the considered FEL model, we will try to show the effect of energy and density separation of REBs on the peak growth and wavenumber of two resonances in the system. The intensity of the maximum undulator magnetic field is 2.91 Kg and the period length of the undulator is assumed 2 cm. The stability properties for the uncoupled wave are plotted in Fig. 2, where the real part of scaled frequency $Re(\omega/ck_w)$, is plotted versus scaled wavenumber (k/k_w), $\gamma_1 = 1.3$, $\gamma_2 = 1.33$, $\hat{\delta}_1 = \hat{\delta}_2 = 0$.

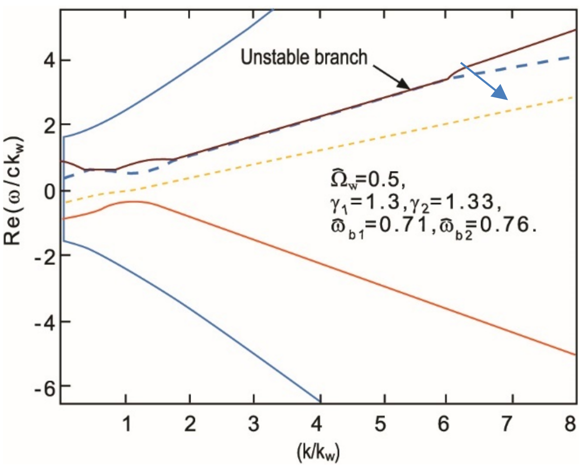

Fig. 2. (Colour online) Plot of frequency versus wavenumber for uncoupled wave for the case of $\hat{\Omega}_w = 0.5$; $\gamma_1 = 1.3$; $\gamma_2 = 1.33$; $\hat{\omega}_{b1} = 0.71$; $\hat{\omega}_{b2} = 0.76$; $\hat{\delta}_1 = \hat{\delta}_2 = 0.0$.

In Fig. 3 peak growth (red) and scaled wavenumber (blue) have plotted vs energy separation of the beams for the FEL resonance band in the base of Eq. (11). The chosen parameters are, $\hat{\Omega}_w = 0.5$, $\gamma_1 = 1.3$, $\gamma_2 = 1.3 - 1.95$, $\hat{\omega}_{b1} = 0.71$, $\hat{\omega}_{b2} = 0.76$, $\hat{\delta}_1 = \hat{\delta}_2 = 0.0$, as seen, increasing the energy separation results decreasing the peak growth and cause to increasing the wave number of the FEL resonance.

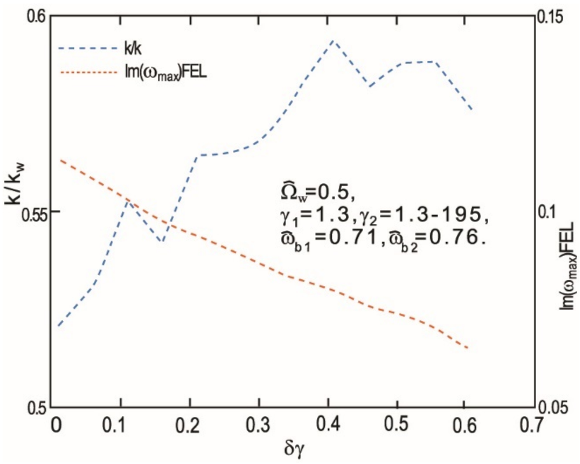

Fig. 3. (Colour online) Peak growth (red) and scaled wavenumber (blue) vs energy separation of electron beams for the case of $\hat{\Omega}_w = 0.5$; $\gamma_1 = 1.3$; $\gamma_2 = 1.3 - 1.95$; $\hat{\omega}_{b1} = 0.71$; $\hat{\omega}_{b2} = 0.76$; $\hat{\delta}_1 = \hat{\delta}_2 = 0.0$.

As predicted by (Aghamir and Mahdizadeh, 2012; Mehdian *et al.*, 2008), in two stream FELs we have two resonance bands, FEL and two stream FEL resonances in Fig. 4 peak growth (red) and scaled wavenumber (blue) for two stream FEL resonance has plotted vs energy separation of the beams in the base of Eq. (11). The chosen parameters are, $\hat{\Omega}_w = 0.5$, $\gamma_1 = 1.3$, $\gamma_2 = 1.3 - 1.95$, $\hat{\omega}_{b1} = 0.71$, $\hat{\omega}_{b2} = 0.76$, $\hat{\delta}_1 = \hat{\delta}_2 = 0.0$. Here, increasing the energy separation results in decreasing the peak growth and causes to increase the wave number of the FEL resonance. The same behaviour has been observed for two beam FEL resonance, as seen in Fig. 4. However, in FEL resonance scaled wavenumber increased to 0.47 and then decreased, while in two stream FEL resonance it increased monotonically.

In Fig. 5, the effect of the REBs density separation on the scaled wavenumber and peak growth of FEL resonance is plotted. As we have seen, with increasing the electron beam density separation the scaled peak growth is increased, while the scaled wavenumber has slightly decreased.

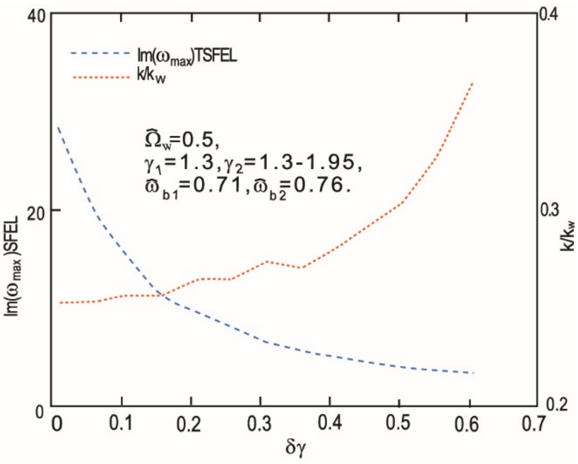

Fig. 4. (Colour online) peak growth (blue) and scaled wavenumber (red) vs energy separation of electron beams for the case of $\widehat{\Omega}_w = 0.5$; $\gamma_1 = 1.3$; $\gamma_2 = 1.3 - 1.95$; $\widehat{\omega}_{b1} = 0.71$; $\widehat{\omega}_{b2} = 0.76$; $\widehat{\delta}_1 = \widehat{\delta}_2 = 0.0$.

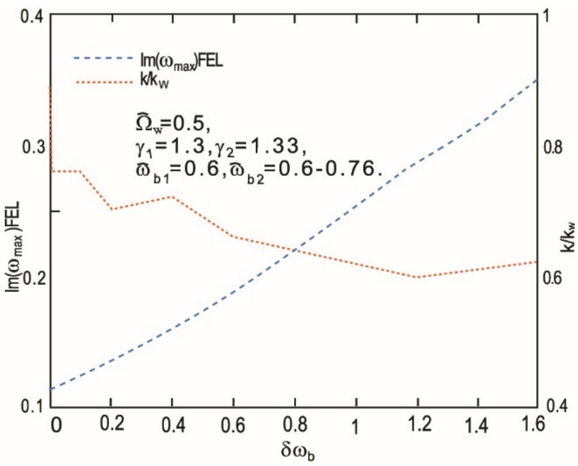

Fig. 5. (Colour online) peak growth (blue) and scaled wavenumber (red) vs energy separation of electron beams for the case of $\widehat{\Omega}_w = 0.5$; $\widehat{\omega}_{b1} = 0.6$; $\widehat{\omega}_{b2} = 0.6 - 0.76$; $\gamma_1 = 1.3$; $\gamma_2 = 1.33$; $\widehat{\delta}_1 = \widehat{\delta}_2 = 0.0$.

Variation of the REB density separation vs the scaled wavenumber and peak growth for two beam FEL resonance has been shown in Fig. 6. With increasing the electron beam density separation, the scaled peak growth is increased, while the scaled wavenumber is decreased.

In two beam FELs one resonance which so, called two stream FEL resonance occurs in the shorter wavelength. In addition, by adjusting energy and density separation (corresponding to the voltage and current separation, respectively) one could make them a promising approach for more tuning laser like radiation.

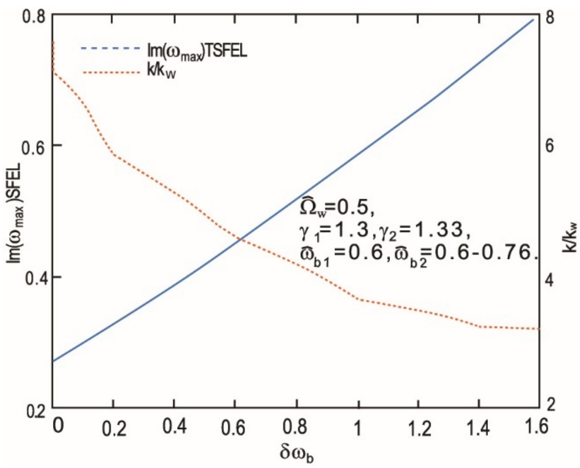

Fig. 6. (Colour online) peak growth (blue) and scaled wavenumber (red) vs energy separation of electron beams for the case of $\widehat{\Omega}_w = 0.5$; $\widehat{\omega}_{b1} = 0.6$; $\widehat{\omega}_{b2} = 0.6 - 0.76$; $\gamma_1 = 1.3$; $\gamma_2 = 1.33$; $\widehat{\delta}_1 = \widehat{\delta}_2 = 0.0$.

It is worth mentioning that, the energy and density separation of REBs vs the peak growths and wavenumbers of the FEL and the two stream FEL resonances have opposite behaviour. More separation of the REBs energy causes to decrease in peak growth of the FEL resonance, while results in to increase in peak growth of the two stream FEL resonance. On the other hand, more separation of the REBs density causes to increase peak growth of the FEL resonance which results in to decrease peak growth of the two stream FEL resonance, which is the same as the outcomes reported by (Mahdizadeh, 2018). It is reasonable because the initial data that we have used for the numerical solution were in the Raman regime (week-undulator). Because the effect of the space-charge potential of the plasma waves in this regime is greater than the effect of the ponderomotive potential.

Conclusion

By the numerical solution (iteration method) the general DR of a FEL model with two REBs is solved. Electron beam energy and density separation effects on the peak growth and the wavenumber at both resonances have been verified in the Raman regime, in the framework of kinetic description. It has shown that increasing the energy separation of two REBs results in decreasing the peak growth of the FEL resonance, while it causes to increase in its wave number. However, it has the opposite behaviour for two stream FEL resonance. Depending on the desired wavelength or power the Figs. (3-6) can be used.

This study has been done in the Raman regime, however in the high-gain Campton regime with different DF such as the Gaussian DF could be an interesting adventure for our future work.

Acknowledgement

Many thanks to the research chancellor of the Sabzevar and Mashhad Branches of Islamic Azad University.

Conflict of Interest. The authors declare that they have no conflict of interest.

References

Aghamir, F.M., Mahdizadeh, N. 2012. Effects of beam temperature and density variation on the growth rate of a two stream free electron laser. *Physics of Plasmas*, **19:** 032114.

Babaei, S., Maraghrchi, B. 2008. Plasma-loaded free-electron laser with the thermal electron beam and background plasma. *Physics of Plasmas*, **15:** 013102.

Davidson, R.C. 2001. *Physics of Non-Neutral Plasmas*, pp. 345, Imperial College Press, London, UK.

Dey, S., Ghosh, S., Maity, D., De, A., Chandra, S. 2022. Two stream plasma instability as a potential mechanism for particle escape from the venusian ionosphere. *Pramana Journal of Physics*, **96:** 213.

Fukui, K., Takada, Y., Sumiyoshi, Imai, T., Takahashi, K. 2006. Infrared multiphoton dissociation spectroscopic analysis of peptides and oligosaccharides by using Fourier transform ion cyclotron resonance mass spectrometry with a midinfrared free-electron laser. *Journal of Physical Chemistry B*, **110** 16111-6.

Goswami, J., Chandra, S., Das, Ch., Sarkar, J. 2022. Nonlinear wave-wave interaction in semiconductor junction diode. *IEEE Transactions on Plasma Science*, **50:** 1508-1517.

Ghosh, A., Goswami, J., Chandra, S., Das, Ch., Arya, Y., Chhibber, H. 2022. Resonant interactions and Chaotic excitation in nonlinear surface waves in dense plasma. *IEEE Transactions on Plasma Science*, **50:** 1524-1535.

Ghosh, S., Saha, S., Chakraborty, T., Sadhukhan, K., Bhanja, R., Chandra, S. 2021. Linear and non-linear properties of electron acoustic waves in a viscous plasma. *The African Review of Physics*, **vol. 15**, pp. 247, ICTP Publication.

Kulish, V.V., Lysenko A.V., Savchenko V.I. 2003. Two-beam free electron lasers: physical and project analysis of the multi harmonically models. *International Journal of Infrared and Millimeter Waves*, **24:** 501-524.

Liu, W., Yang, Z., Liang, Z. 2006. Instability of two-beam free-electron laser with an axial guiding magnetic field. *International Journal of Infrared and Millimeter Waves*, **27:** 1073.

Mahdizadeh, N., Aghamir, F.M. 2013. Effects of the finite beam and plasma temperature on the growth rate of a two beam free electron laser with background plasma. *Journal of Applied Physics*, **113:** 083305.

Mahdizadeh, N. 2018. The effects of density and energy separation of two beams on the picks of growth rate in a two stream free electron laser. *Quarterly Journal of Optoelectronic*, **2:** 45-52.

McNeil, B.W.J., Robb, G.R.M., Poole, M.W. 2004. Two beam free electron laser. *Physical Review E*, **70:** 035501(R).

Mehdian, H., Saviz, S., Hasanbeigi, A. 2008. Two-beam instability in free electron lasers with a planar undulator and an axial guide magnetic field. *Physics of Plasmas*, **15:** 043103.

Mukhopadhyay, A., Bagui, D., Chandra, S. 2020. Electrostatic shock fronts in two component plasma and its evolution into Rogue wave type solitary structures. *The African Review of Physics*, **15:** 1521.

Ogi, Y., Kawashita, M., Tsukiyama, K. 2006. Vibrational excitation of the *v*2 mode of NH_3 by infrared free electron laser. *Chemical Physics Letters*, **421:** 96-100.

Razaghzadeh, M., Mahdizadeh, N., Haghpeima, A., Haghighi, B. 2023. Dispersion formula of a free electron laser with two relativistic electron beams and water bag distribution function. *Laser Physics, IOP Publishing*, **33:** 106202.

Rouhani, M.H., Maraghechi, B., Saberi, H. 2009. Efficiency enhancement of a two beam free-electron laser. *Physics of Plasmas*, **16:** 123105.

Saldin, E.L., Schneidmiller, E.A., Yurkov, M.V. 2000. *The Physics of Free Electron Lasers*, pp. 457, Springer-Verlag Berlin, Heidelberg, New York, USA.

Sarkar, J., Chandra, S., Goswami, J., Das, C., Ghosh, B. 2021. Growth of RT instability at the accreting magnetospheric boundary of neutron stars. In: *Proceedings of AIP Conference*, **2319,** pp. 30006, AIP Publishing.

Sarkar, J., Chandra, S., Goswami, J., Ghosh, B. 2022. Heliospheric two stream instability with degenerate electron plasma. In: *Nonlinear Dynamics and Applications*, Banerjee, S., Saha, A. (eds), pp. 25-42, Springer, Cham.

Vlasov, A.A., 1938. On vibration properties of electron gas. *Journal of Experimental and Theoretical Physics*, **8:** 291.