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Multifractal Analysis and Interpretation of Reservoir Fluids

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Abstract. Spectral decomposition technique is used in seismic interpretation for identifying hydrocarbon reservoirs, thin beds and channels. Seismic traces contain discontinuities, also known as singularities, which can be used to derive important geological and geophysical information about the earth formation. To analyse these singularities, statistical techniques are necessary to detect variations in rock and fluid properties. The current study describes a method based on characteristics of singularity spectrum derived from analysis of multifractals for detecting reservoir homogeneity (same reservoir fluids) or heterogeneity (different reservoir fluids) on synthesized and modelled seismograms. Singularity spectrum features namely correlation dimension and width were investigated. By using these additional characteristics, prospective hydrocarbon zones become more reliable, reducing drilling uncertainty and possibly improving reservoir fluid flow modelling.

Keywords: reservoir modelling, multifractals, spectral decomposition, wavelet transform

Introduction

The significant temporal variation in the spectral content of seismograms require unconventional decomposition methods for analysis. In 1990s, the theory of wavelet was developed, which has since been utilized in numerous fields of data analysis, including the petroleum industry. Spectral decomposition techniques for seismic interpretation have revolutionized oil and gas exploration over the past decade.

Spectral decomposition techniques have made significant strides in seismic interpretation for hydrocarbon exploration over the past ten years. The spectral decomposition method is a technique used for temporal-spectral analysis of signals. In the petroleum industry, many spectral decomposition techniques such as continuous wavelet transform, cepstral transform, matching pursuit decomposition, wigner-ville distribution and have been used to enhance processing algorithms, including the estimation of thin beds, channel thickness and tuning hydrocarbon reservoirs. The primary objective of these methods is to improve the accuracy of detecting hydrocarbon zones, thereby reducing drilling uncertainties. By enhancing the interpretation of seismic data, reservoir engineers can make better decisions regarding production plans (Marianne and Micheal, 2006; Satish *et al.*, 2005; John *et al.*, 2003; Avijit and David, 1995).

In the research study by (Mela and Louie, 2001), the author found that the spatial variability of porosity and fracture content in reservoirs can be characterized using statistical parameters such as correlation length and fractal dimension computed from seismic data through variograms and power spectra analysis. They found that these parameters play a critical role in fluid flow modeling of reservoirs. Although mapping permeability required further exploration. (Sandhya and Cohen, 2004), used singularity spectrum to identify changes in lithology and pore fluid types on both model studies and real seismic data in their study. Whereas (Zhang *et al.*, 2020) used multifractal analysis to characterize the heterogeneity of pore space in deep-buried dolomite. Such additional parameters hence can be useful in improving reservoir exploitation and estimating reservoir properties.

Appropriate tools are essential to accurately analyse homogeneity or heterogeneity of reservoir fluids (RF). The current study describes the process of multifractal analysis used to obtain useful fractal parameters and generate a singularity spectrum. The use of singularity spectrum attributes for analyzing the homogeneity or heterogeneity of reservoirs through the analysis of well log seismograms and geological modelling is also discussed, with the aim of facilitating more accurate reservoir simulation.

Singularity spectrum. Variation in the geology and geophysics of the earth's formation can cause

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discontinuities, or singularities, in certain attributes of a seismic trace (Rai *et al*., 2020). These singularities are highly informative, representing rapid changes in variable values over very small increments of time or position. They can be observed at points where the time expansion of a signal contains components at fractional powers of time. Multifractal analysis (MFA) is a statistical technique for characterising a signal's singularity content, either locally or globally. MFA parameters include fractal dimension (FD), holder exponents (HE) and singularity spectrum (SS) (Lopes and Betrouni, 2009). By using MFA to data, it is possible to identify and classify various change in states within the signal (Boulassel *et al.*, 2021).

SS is a valuable analytical tool for examining the overall singularity variation in a signal, providing information on the distribution of HE. Several approaches have been developed (Antonia *et al.*, 2005) to compute the SS, including the moment method, wavelet transform modulus maxima method (WTMM), the gradient histogram method, gradient modulus wavelet projection (GMWP) method, wavelet leader-based multifractal analysis (WL) and multifractal de-trended fluctuation analysis (MFDA) method (Alam *et al*., 2023; Amoura *et al*., 2022).

SS has emerged as a robust tool for analyzing time series data across diverse fields such as biology (Westra, 2002), physics, economics and technical sciences (Khan *et al*., 2006). It has been applied in various applications of health sciences (Stosic and Stosic, 2006) and engineering domain (Alam *et al*., 2023; Enescu, 2004; Barry and Kisner, 2004).

The two key parameters used for classification in SS analysis are the width ($\Delta\alpha=\alpha_{max}-\alpha_{min}$) and correlation dimension (C_d). The width is a measure of the signal's multifractality, where a wider spectrum indicates greater complexity. Mathematically, it is calculated by subtracting the strength of the strongest singularity from that of the weakest. The singularity spectrum also provides a set of generalized fractal dimensions, including Hausdorff, information and correlation dimensions. As mentioned by (Grassberger and Procaccia, 1983), C_d measures set of random point's dimensionality.

This study aims to utilize SS to identify the parameters that can aid in detecting homogeneity in the RF. Wavelet transform is selected as the appropriate method for identifying discontinuities and irregularities due to its time-frequency localization property. The wavelet transform modulus maxima (WTMM) method is applied on seismograms and SS analysis is performed across various conditions.

Material and Method

SS is used on synthetic seismograms generated from earth models and well logs in this work. The characteristics that affect the SS are specifically investigated. The computation of SS based on WTMM consists of the following basic steps described in Fig. 1. Further details of it can be found in (Staal, 1995; Mallat and Hwang, 1992).

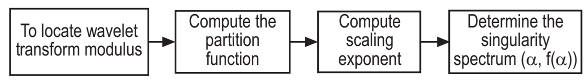

Fig. 1. Process of singularity spectrum.

Step I is to locate the wavelet transform modulus maxima lines, which is performed by applying wavelet transform to a function f(t) with time parameter t to a function f(σ,τ) depends on a scaling attribute σ and a temporal attribute τ with an appropriate analysing wavelet ψ. This is expressed mathematically in Equation 1 reported by (Mallat and Hwang, 1992).

$$f(\sigma,\tau) = W\{f,\psi\}(\sigma,\tau) = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{|\sigma|}}\overline{\psi\left(\frac{t-\tau}{\sigma}\right)}dt \;.........(1)$$

The local maxima of the absolute value of the WT as a function of position at each scale is then found such that:

A local extrema of $W\{f,\psi\}(\sigma_0,\tau)$ is that point (σ_0,τ_0) such that $\frac{dW\{f,\psi\}(\sigma_0,\tau)}{d\tau}$ has a zero crossing at $\tau=\tau_0$ when τ varies.

A modulus maxima is any point (σ_0,τ_0) such that $|W\{f,\psi\}(\sigma_0,\tau)|<|W\{f,\psi\}(\sigma_0,\tau_0)|$ when τ belongs to either the right or left of the neighbourhood of τ_0 and $|W\{f,\psi\}(\sigma_0,\tau)|\leq|W\{f,\psi\}(\sigma_0,\tau_0)|$ when τ_0 belongs to the other neighbourhood of τ_0.

Modulus maxima line is then any connected curve in the scale time plane along which all points are modulus maxima. Next, we track maxima lines for increasing scale σ by choosing at each scale the maximum between all previous values at smaller scales $\sigma_0<\sigma$. By this way, we replace the WT coefficient extremum value by the maximum value along its line. Hence a modulus maxima line is identified (Mallat and Hwang, 1992).

Step II involves the computation of the partition function $Z(\sigma,q)$ in which we sum the maximum values of the WT along the modified maxima lines to get the partition function $Z(\sigma,q)$ by using the Equation 2. The partition function $Z(\sigma,q)$ captures the statistical behaviour of the coefficients at different moment orders. The moment q therefore accentuates different aspects of the underlying dynamical process (West and Grigolini, 2011). For $q>0$, the partition function $Z(\sigma,q)$ emphasizes large fluctuations and strong singularities, whereas for $q<0$, the partition function stresses the small fluctuations and the weak singularities. Thus, partition function measures the scaling of moments and high order dependencies of wavelet coefficients and the singularity structure all in one (Staal, 1995).

$$Z(\sigma,q)=\frac{1}{\sqrt{\sigma}}\sum_{m}(\sup \mathrm{WTMML}(m))^{q} \qquad (2)$$

Step III is to compute the scaling exponents $\tau(q)$ or mass exponent which is obtained as the slope of the log-log plots of $Z(\sigma,q)$ versus σ by linear regression. It describes the statistical moments. For a monofractal signal $\tau(q)$ is a straight line (Staal, 1995). This is mathematically shown in Equation 3.

$$\tau(q)=\lim_{\sigma\to 0}\frac{\log Z(\sigma,q)}{\log \sigma} \qquad (3)$$

Finally **Step IV** involves computing the singularity spectrum (α, $f(\alpha)$) from the $\tau(q)$ through the Legendre transform shown in Equation 4 and 5 where α is the holder exponent and $f(\alpha)$ is the fractal dimension. $f(\alpha)$ of a MF signal is similar to an inverted parabola (Staal, 1995).

$$\alpha=\frac{d\tau(q)}{dq} \qquad (4)$$

$$f(\alpha)=q\frac{d\tau(q)}{dq}-\tau(q) \qquad (5)$$

From the singularity spectrum obtained we can extract attributes such as width and correlation dimension. Width is mathematically expressed as

$$\Delta\alpha=\alpha_{max}-\alpha_{min} \qquad (6)$$

Note: that α_{max} is the maximum value of singularity and α_{min} is the minimum value of singularity. C_d is the value of fractal dimension at moment order q=2. Thus all the synthetic seismograms from earth models and well logs are analyzed by following the series of steps described above.

Earth model seismograms. Seismograms for three basic earth models-shale-oilsand, shale-gassand and shale-waters and which is first created. Matlab software is used to create the modelled seismic traces at a sampling frequency of 50 Hz and a Ricker wavelet with a central frequency of 60 Hertz. The rocks with RF (oil, gas) shown in Table 1's density velocity values are used. The Gaussian's first derivative at moment orders q = [-6, 6] is used to analyse the data at a frequency range of 20-80 Hz in order to identify low and high irregularities, respectively. Using the FracLab programme (FracLab, 2006), the correlation dimension and width of the singularity spectrum are determined and documented. The three scenarios listed below were used to examine and analyse the singularity spectra of the earth model seismo grams.

Geological model. The layers of the geological structures are represented by the (Geological Model). For the different RF, the impact of layers {4, 6, 8, 12} with a fixed thickness (0.045 Km) on the SS was examined. With varying numbers of layers, it was found that the values of singularity spectrum attributes barely altered. The singularity spectrum of seismic tracks with gas as a RF is shown in Fig. 2(a) at various stratigraphic levels. The analysed gas traces' singularity spectrum characteristics are tallied and displayed in Table 2. As can be seen, the $\Delta\alpha$ and C_d of the spectrum vary very little as the stratigraphy changes. We can observe a comparable pattern in the seismic traces for oil, as shown in Fig. 2(b), along with corresponding characteristics are presented in Table 2.

Thickness. Earth's layers occur naturally in varying thicknesses. To investigate this, seismograms for a 14-layered earth model were created with thicknesses of {0.04, 0.05, 0.06} kms and the SS and its attributes were evaluated. Figure 3 illustrates the singularity spectra of gas and oil seismic traces for different

Table 1. Density and velocity of rocks

	Velocity (Km/sec)	Density (g/cc)
Shale	2.94	2.38
Gas sand	2.5	1.8
Oil sand	2.8	2.1

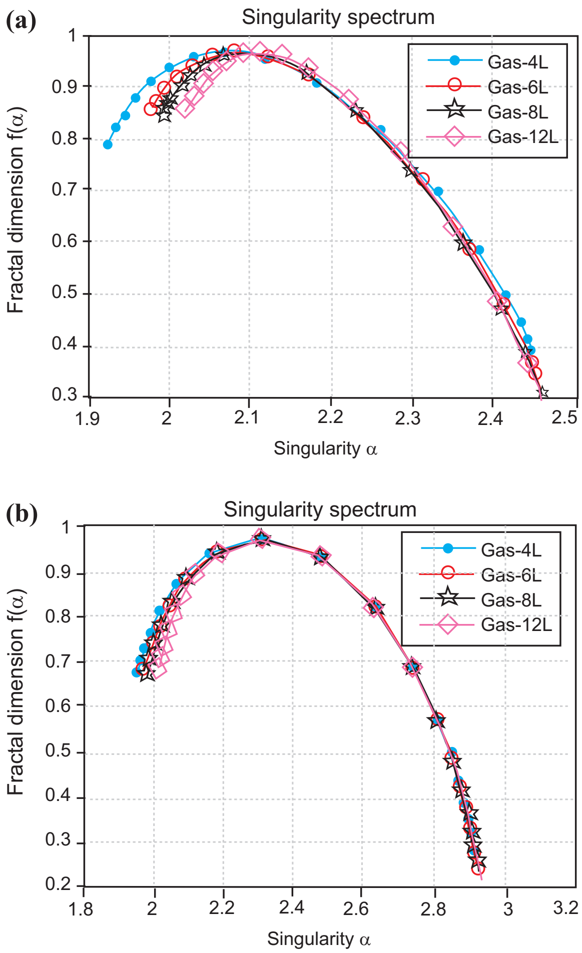

Fig. 2. Singularity spectra of different stratigraphic based seismic traces, **(a)** gas-based reservoir, **(b)** oil-based reservoir.

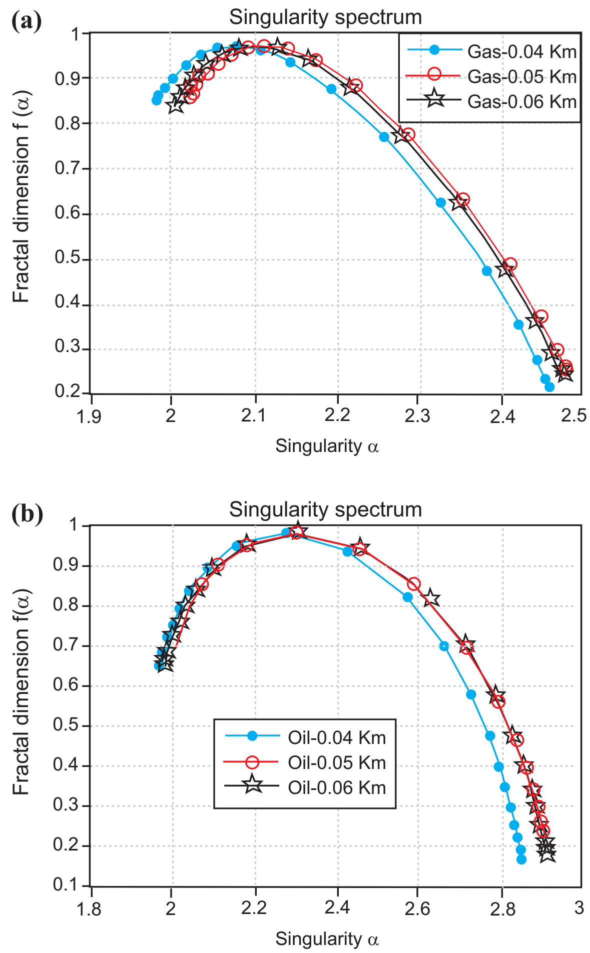

Fig. 3. Singularity spectra of different stratigraphic thickness based seismic traces traces **(a)** gas-based reservoir **(b)** oil-based reservoir.

Table 2. Effect of stratigraphy on singularity spectrum attributes for a gas and oil-based reservoir with varying stratigraphic layers.

Stratigraphic layers	Gas reservoir		Oil reservoir	
	C_d	Δα	C_d	Δα
4.0	0.9100	0.5200	0.8000	0.9580
6.0	0.9300	0.4700	0.8100	0.9480
8.0	0.9340	0.4600	0.8200	0.9480
12.0	0.9380	0.4500	0.8380	0.9200

Table 3. Singularity spectrum features of seismic traces across different thickness for a gas and oil-based reservoirs.

Stratigraphic thickness (m)	Gas reservoir		Oil reservoir	
	C_d	Δα	C_d	Δα
40	0.9350	0.4500	0.8300	0.9000
50	0.9360	0.4730	0.8380	0.9200
60	0.9380	0.4700	0.8200	0.9572

thicknesses. Despite these variations, the singularity spectrum attributes remained relatively constant, as evident from Table 3.

Reservoir fluid. Examining the singularity spectra and its variations resulting from different RF such as oil and gas is a critical aspect. To accomplish this, seismic traces were generated for the respective RF, considering distinct stratigraphy and thicknesses. Figure 4 illustrates the SS for the different RF in the generated Seismic model for an 8-layered stratigraphic model. It can be observed that the gas reservoirs have a narrower width compared to the oil reservoirs. Similar observation is seen across the singularity spectral parameters in earth modelled seismograms for various stratigraphy and thickness in Table 2 and 3. Notably, the higher C_d of gas indicates its irregularity, highlighting the potential usefulness of the singularity spectrum in hydrocarbon detection and delineation through seismic traces.

Well log simulation. Sonic and density logs of four distinct wells (WA, WB, WC and WD) with confirmed RF were collected. Using Syntool™ (SynTool, 2006), a well log seismogram is produced at a sample frequency of 50 Hz using the negative normalized second derivative of a Gaussian function, with a peak frequency of forty Hertz. There is also stratigraphical correlation between the well log seismograms. In the current study, a uniform or homogeneous well is considered as a well with one reservoir fluid but a heterogeneous well contains multiple reservoir fluids. WA has oil where gas was present in both WB, WC and WD has both RF. Because the method employed low sampling frequency, each detected hydrocarbon zone is estimated to comprise only 3-4 data samples, with each sample representing an 8-foot formation. To ensure an adequate data size for the analysis of SS, the detected zones are upsampled by 20. The SS of these zones are then evaluated at a range of [20, 80] Hz using first derivative of gaussian. The same moment order range is used.

The SS of the homogeneous reservoirs in the WB and stratigraphically correlated wells (WA and WD) are depicted in Fig. 5(a,b), while Fig. 6(a,b) illustrate the SS of the heterogeneous reservoirs in the WD and stratigraphically correlated wells (WA, WB, WD). The SS attributes are presented in Tables 4 and 5. The results indicate that the values of spectral parameter vary with change in RF, underscoring the usefulness of this technique in detecting the similar RF in a zone.

Results and Discussion

Earth model seismograms. ***Geological model.*** The analysis of Table 2 reveals that there is a negligible variation in α and C_d of both RF as the stratigraphic layers increase. Although the count of transitions increases with the increasing number of layers, the transient type remains constant, leading to the same set

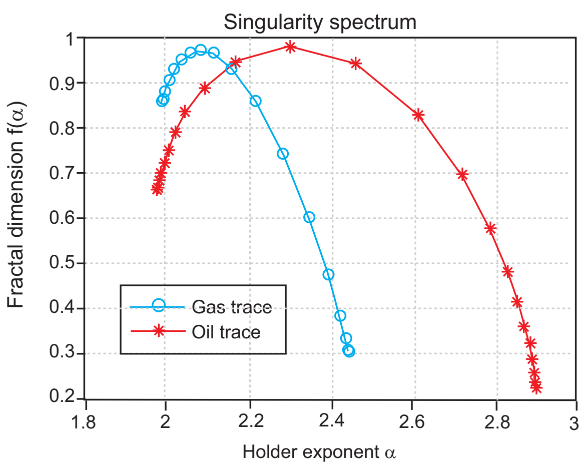

Fig. 4. Singularity spectra of different reservoir fluids seismic traces for an 8-layer stratigraphy of 0.045 Km thickness.

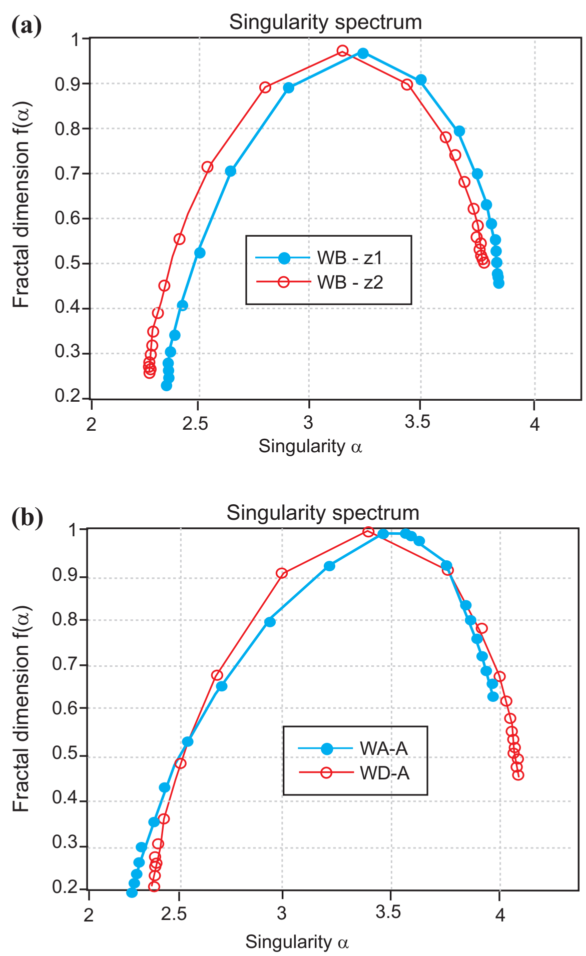

Fig. 5. Singularity spectrum of homogeneous reservoirs across **(a)** vertical well WB with two zones z1, z2 of gas reservoir and **(b)** a stratigraphic correlated well WA and WD with oil reservoir

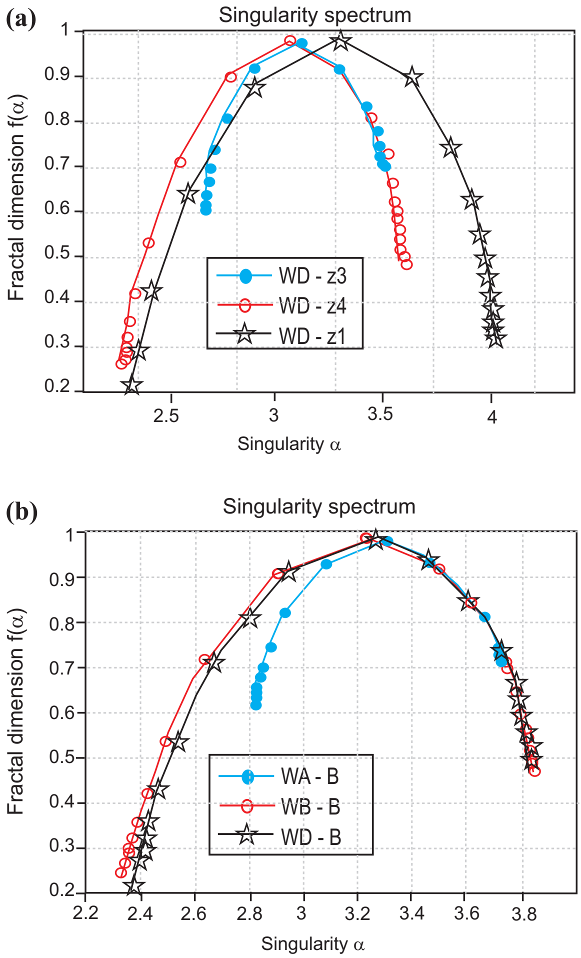

Fig. 6. Singularity spectrum of heterogeneous reservoirs across **(a)** vertical well WD with three zones z1, z3 and z4 and **(b)** a stratigraphic correlated well WA, WB and WD, with the presence of both oil and gas reservoir fluid.

of HE values. This suggests that, for the same RF, the SS behavior remains consistent as the number of layers increases. In essence, this implies that the SS is not dependent on the stratigraphic layers with similar RF.

Thickness. Based on the analysis of Table 3, one can observe that the SS attributes exhibit negligible changes as the thickness increases. With increasing thickness, a similar transient occurs. Therefore, within the sampled window, the number of transients remains constant. The implication of this discovery is that the SS remains unaffected by the thickness of the reservoir, indicating its independence on this parameter. However, it should be noted that testing smaller thicknesses, such as 1 m, 5 m, 10 m and 20 m, could provide further insights. Thus, by employing a consistent window length, it is possible to compare and measure SS attributes with a high degree of reliability.

Table 4. Singularity spectrum parameters of the well section mentioned in Fig. 5, depicting a homogenous presence of reservoir fluid.

Gas samples	C_d	$\Delta\alpha$	Oil samples	C_d	$\Delta\alpha$
WB-z1	0.7200	1.4830	WA-A	0.6490	1.6700
WB-z2	0.7300	1.4800	WD-A	0.6690	1.6800

Table 5. Singularity spectrum parameters of the well section mentioned in Fig. 6, depicting a heterogeneous presence of reservoir fluid.

Samples	C_d	$\Delta\alpha$	Samples	C_d	$\Delta\alpha$
WD-z1 (Gas)	0.6100	1.8600	WA-B (Oil)	0.7390	0.9030
WD-z3 (Oil)	0.7400	0.900	WB-B(Gas)	0.5300	1.4800
WD-z4 (Gas)	0.6500	1.6800	WD-B(Gas)	0.5300	1.4200

Reservoir fluid. Figure 4 shows the singularity spectra for an oil and gas trace with an 8-layered stratigraphy of 0.045 Km constant thickness. We can see that the singularity spectra can be used as a possible parameter for hydrocarbon delineation. It was observed that gas seismic traces contain strong transients i.e. (stronger singularities) compared to oil seismic traces, resulting in lower values of i.e. (αmax=2.46). On the other hand, the presence of weaker singularities in oil seismic traces leads to higher values of i.e. (αmax=2.9), overall resulting in wider α of the oil. It is common to observe strong singularities in gas traces due to the sharp variations they exhibit, which is why gas occupies higher C_d values than oil. Similarly, from Table 2 we can conclude that with changing stratigraphy and constant thickness, oil reservoir attains higher width $\Delta\alpha$ and lower C_d as compared to gas reservoir which exhibits lower $\Delta\alpha$ and higher C_d. We also derive the same conclusion from Table 3 which represents spectral parameters across changing thickness for a constant 14-layer stratigraphy.

The analysis of the singularity spectrum and its attributes can provide valuable information on the behaviour of seismic traces and can help in identifying the presence of oil and gas reservoir fluids. Therefore, the development of an analysis tool based on the singularity spectrum can be useful for the delineation of these fluids.

Well log seismo-grams. The SS attributes, such as C_d and $\Delta\alpha$, show a continuous change in value for homogeneous oil and gas RF in both vertical and stratigraphic correlated wells, as seen in Table 4. This continuity indicates that SS analysis can be used to identify the homogeneity of RF in seismic sections. Interestingly, similar SS and attributes are observed for the same reservoir fluid, despite existing at different stratigraphic levels and thicknesses in the analyzed wells. This finding suggests that SS analysis is independent of stratigraphy and thickness in well log seismograms.

Table 5 reveals that for heterogeneous vertical wells/ stratigraphic correlated wells, the singularity spectrum attributes (correlation dimension, width) exhibit abrupt changes. Such changes suggest variations in the properties of the reservoir fluid. For instance, in Table 5, the correlation dimension decreases from 0.739 to 0.53 as we move from oil to gas zone, and then remains constant at 0.53 when we hit another gas zone. Thus, SS analysis can be employed as a potential tool for accurately classifying and delineating RF, facilitating their modeling.

Conclusion

The study highlights the significance of SS analysis in identifying the homogeneity of reservoir fluids through seismic modeling and well log seismograms. By analyzing the various seismic earth models, the study revealed that SS behaviour is only sensitive to changes in the reservoir fluid and not influenced by stratigraphy and thickness. Even with changing stratigraphy and increasing thickness, occurrence rate did not change, indicating the independence of the SS. However, the SS attributes, such as α and C_d, showed a significant difference with changing reservoir fluids. The seismic trace of gas contains strong transients resulting in a low width α and high fractal dimensions, while oil seismic trace has weaker singularities, resulting in wider width and lower fractal dimension values. This finding emphasizes the importance of singularity spectrum analysis in accurately delineating and classifying reservoir fluids.

In addition to the existing detection methods, the SS parameters can be utilized to identify the consistency of the reservoir fluids. However, these parameters should be complemented with other analytical techniques to create precise models of the fluid flow in the reservoir. Thorough seismic data analysis is critical for improving the oil and gas industry's performance by enabling efficient well placement, reducing drilling uncertainties, and improving the accuracy of seismic volume interpretation.

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Conflict of Interest. The authors declare they have no conflict of interest.

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