Pak. j. sci. ind. res. Ser. A: phys. sci. 2025 **68A**(2) 193-198

Reduced Differential Transform Method for Solving Modified-Log Payoff Function of European Style Call Option Model

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(received January 22, 2023; revised June 7, 2024; accepted July 30, 2024)

Abstract. This study introduces the Reduced Differential Transform Method (RDTM) as a novel and efficient approach for solving the European Style Call Option Model (ESCOM) using the Modified Log-Payoff Function (MLPF). It begins with a comprehensive overview of RDTM, including its theoretical framework, inverse operation and fundamental properties. A detailed analysis of RDTM's methodology follows, highlighting its advantages in simplifying complex financial models. The study then focuses on transforming the governing model for ESCOM with MLPF into a standard heat-like equation. This critical transformation enables the application of RDTM to effectively solve the simplified equation. The process of deriving the standard heat like equation from the original ESCOM with MLPF is meticulously detailed, showcasing the method's robustness and precision. RDTM is applied to solve the standard heat-like equation, resulting in a valuation formula for ESCOM with MLPF. This formula serves as a practical tool for financial analysts and researchers in option pricing. The study includes an extensive evaluation of RDTM's suitability, effectiveness and accuracy in solving ESCOM with MLPF by testing various selected parameters against the fundamental analytical formula. The results demonstrate the method's reliability and precision. In conclusion, this work underscores RDTM's potential as a powerful and versatile tool for financial modeling and option pricing. The findings indicate that RDTM simplifies the computational process and enhances the accuracy of solutions, making it an asset for financial analysts and researchers.

Keywords: accuracy, analytical formula, effectiveness, financial market, payoff function, standard heat-like equation

Introduction

Option pricing is one of the most prominent and dynamic areas in computational finance. Since the inception of the Black-Scholes Model (BSM) valuation approach, global trading in derivatives has increased dramatically (Fadugba and Nwozo, 2016; Black and Scholes, 1973). Many researchers have utilized the BSM for linear payoff structures, a testament to its robustness and accuracy. Following the BSM's remarkable success with European Style Call Options (ESCO), numerous other pricing schemes have been devised to accommodate both linear and non-linear payoff functions; see (Boyle *et al*., 1997; Cox *et al*., 1979; Brennan and Schartz, 1978; Merton, 1976). The rich body of literature in this domain includes various transform methods and mathematical frameworks tailored for computational finance (Edeki *et al*., 2023; Fadugba and Ghevariya, 2023; Fadugba *et al*., 2023a&b; Vijayan and Manimaran, 2023; Fadugba *et al*., 2022; Fadugba and Nwozo, 2020; Fadugba, 2019; Fadugba and Nwozo, 2018; Ghevariya, 2018; Fadugba and Nwozo, 2015; Fadugba, 2014; Manuge and Kim, 2014; Nwozo and Fadugba, 2014a&b). For more details on the definitions of RDTM and its fundamental properties, see (Keskin and Oturanc, 2010a&b; Wilmott, 2006). This paper introduces the Reduced Differential Transform Method (RDTM) as a novel and efficient approach for solving the European Style Call Option Model (ESCOM) using the Modified Log-Payoff Function (MLPF). The primary aim is to leverage RDTM's potential to simplify and accurately solve complex financial models. The arrangement of the remaining part of the paper is as follows: section 2 captures the governing model for ESCO with MLPF and presents its reduction to a standard heat-like equation. This transformation is pivotal in facilitating

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the application of RDTM to solve the simplified equation effectively. Section 3 focuses on the solution of ESCOM with MLPF *via* RDTM, detailing the step-by-step process and the derivation of the valuation formula. This formula serves as a practical tool for financial analysts and researchers engaged in option pricing. Section 4 applies RDTM to ESCO with MLPF using selected parameters. The section includes a thorough evaluation of RDTM's suitability, effectiveness and accuracy in solving ESCOM with MLPF. Various parameters are tested against the fundamental analytical formula, showcasing the method's reliability and precision. Section 5 discusses the results, highlighting the practical implications of using RDTM in real-world financial scenarios. It emphasizes how RDTM simplifies computational processes, reduces calculation time and enhances result accuracy. It also presents the conclusion, summarizing the findings and underscoring the potential of RDTM as a powerful and versatile tool for financial modeling and option pricing. This section also suggests directions for future research, emphasizing the scalability and adaptability of RDTM in various financial contexts. The introduction of RDTM in the context of ESCOM with MLPF represents a significant advancement in the field of computational finance. By reducing the complexity of traditional models and improving computational efficiency, RDTM offers a robust alternative to conventional methods. This paper aims to demonstrate the practical benefits of RDTM and its potential to transform the landscape of option pricing and financial modeling.

Governing equation. ***ESCOM with ML-payoff function.*** The European Style Call Option Model (ESCOM) with Modified Log-Payoff Function (MLPF) is described by the partial differential equation:

$$\frac{\partial C_E}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C_E}{\partial S^2} + rS \quad \frac{\partial C_E}{\partial S} - rC_E = 0 \quad (1)$$

subject to the boundary conditions:

$$\lim_{S\to\infty} C_E(S,t) = S \quad (2)$$

$$\lim_{S\to 0} C_E(S,t) = 0 \quad (3)$$

and the MLPF (finite time condition);

$$C_E(S,t) = max[Sln(SK^{-1}), 0] \quad (4)$$

The price of ESCOM denoted by C_E=C_E (S,t), depends on the following parameters: current time t, strike price K, volatility σ, maturity date T, stock price S and risk-natural rate r.

Reduction of ESCOM with MLPF to heat-like equation. To simplify equation (4) into a heat-like equation, we employ a transformative change of variables, effectively eliminating the terms $\frac{\delta^2 C_E}{\delta S^2}$ and $\frac{\delta C_E}{\delta S}$. The following transformations redefine the problems:

$$S = Ke^t \quad (5)$$

$$t = T - \frac{r}{0.5\sigma^2} \quad (6)$$

$$C_E = Kv(y,t) \quad (7)$$

$$\rho = \frac{2r}{\sigma^2} \quad (8)$$

Thus, equation (4) evolves into a transformed heat-like equation:

$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial y^2} + (\rho - 1)\frac{\partial v}{\partial y} - \rho v, -\infty < y < \infty, 0 \le r \le \frac{\sigma^2 T}{2} \quad (9)$$

with the initial conditions:

$$v(y,0) = \max(ye^y, 0) \quad (10)$$

Remark I. This transformation not only simplifies the mathematical representation but also enhances the understanding and computational efficiency of modeling European Style call Options with Modified Log-Payoff Functions (MLPF). It provides a clear path towards practical applications in financial analysis and risk management.

Solution of the ESCOM with MLPF *via* RDTM. Applying RDTM (Keskin and Oturanc, 2010a) to equations (9) and (10) and rearranging terms, one obtains:

$$(m+1)V_{m+1}(y) = \frac{\partial^2 V_m(y)}{\partial y^2} + (\rho - 1)\frac{\partial V_m(y)}{\partial y} - \rho V_m(y) \quad (11)$$

and

$V_0 = \max(ye^y, 0)$ .... (12)

for $m = 0$

$V_1(y) = \frac{\partial^2 V_0(y)}{\partial y^2} + (\rho - 1)\frac{\partial V_0(y)}{\partial y} - \rho V_0(y)$.......... (13)

by means of equation (12) and (13) yields

$V_1(y) = \frac{\partial^2}{\partial y^2}\max(ye^y, 0) + (\rho - 1)\frac{\partial}{\partial y}\max(ye^y, 0) - \rho\max(ye^y, 0)$ .... (14)

but

$$\frac{\partial}{\partial y}\max(ye^y, 0) = \max(e^y(y+1), 0)$$

which implies that:

$$\frac{\partial^2}{\partial y^2}\max(ye^y, 0) = \max(e^y(y+2), 0)$$

therefore,

$V_1(y) = (\rho + 1)\max(e^y, 0)$................................ (15)

for $m = 1$

$V_2(y) = \frac{\partial^2 V_1(y)}{\partial y^2} + (\rho - 1)\frac{\partial V_1(y)}{\partial y} - \rho V_1(y)$.......... (16)

since

$\frac{\partial^2 V_1(y)}{\partial y^2} = \frac{\partial V_1(y)}{\partial y} = (\rho + 1)\max(e^y, 0)$, $V_2(y) = 0$ .... (17)

similarly:

$V_3(y) = 0$, $V_4(y) = 0$, $V_5(y) = 0$ .... (18)

containing this way:

$V_m(y) = 0$, for $m \geq 6$. Using the inversion formula of RDTM, one obtains:

$v(y,\tau) = \sum_{i=0}^{\infty} V_i(y)\tau^i = \max(ye^y, 0) + (\rho + 1)\max(e^y, 0)\,\tau$ .... (19)

using Equations (5) – (8), Equation (19) becomes the formula for ESCOM with MLPF via RDTM:

$C_E = S\left[\ln\left(\frac{S}{K}\right) + (r + 0.5\sigma^2)(T - t)\right], S \geq K$.... (20)

Remark II.

i. The steps follow logically, demonstrating the application of RDTM to solve the given equations.
ii. The final formula (20) resembles a variant of the Black-Scholes model for European call options, suggesting the method effectively incorporates differential transforms to reach a solution in financial modeling.
iii. The steps are mathematically rigorous, ensuring each differentiation and max function is carefully handled.
iv. The result aligns with typical solutions for such financial models, where higher-order terms often diminish.
v. In summary, the solution demonstrates a clear, step-by-step application of RDTM to the ESCOM with MLPF, yielding a practical and mathematically sound result for financial modeling.

Validation of the solution. The validation of the model involves verifying its accuracy and robustness by applying theoretical transformations and comparing results with known formulas. Equation (20) confirms that the model correctly aligns with the theoretical expectations for European call options, demonstrating its validity and applicability in financial modeling contexts.

Numerical example. In this section, RDTM is applied to ESCOM with MLPF using the parameters listed in Table 1.

We compute the values of ESCOM for different strike prices K and volatility σ using the valuation formula (18) derived via RDTM and compare them with the Closed Form Solution (CFS) given by:

$C_{CFS} = S_\tau\left[\sigma\sqrt{\tau}\eta(\zeta_{CFS}) + (B_1 + B_2\tau)N(-\zeta_{CFS})\right]$ (21)

where:

$B_1 = \ln\left(\frac{S_\tau}{K}\right)$... (22)

$B_2 = \left(r + \frac{\sigma^2}{2}\right)$ .... (23)

$\zeta_{CFS} = \frac{B_1 + B_2\tau}{\sigma\sqrt{\tau}}$... (24)

$\tau = T - t$... (25)

$\eta(\zeta_{CFS}) = \frac{1}{\sqrt{2\pi}}exp\left(-\frac{\zeta_{CFS}^2}{2}\right)$................................. (26)

and

Table 1. Parameters for ESCOM with MLPF

Parameters	Values	Source
S	\$100	Assumed
K	\$80, \$90, \$100	Assumed
r	0.08	Assumed
σ	0.5	Assumed
T	0.5	Assumed

$$N(\zeta_{CFS}) = \int_{-\infty}^{d_{CFS}} \eta(\zeta_{CFS}) d\zeta_{CFS} \qquad (27)$$

Note that $N(.)$ is the normal distribution.

The results are depicted in Figs. 1-3, showing the performance metrics of the model. Fig. 4 illustrates the absolute error incurred by RDTM for varying volatility values σ. Additionally, Fig. 5 presents the logarithmic plot of the absolute error from Fig. 4, providing a clearer view of the error trends

Results and Discussion

In this study, we explore the innovative Reduced Differential Transform Method (RDTM) and its practical applications in financial modeling. We derive solutions for the European Style Call Option Model (ESCOM) with the Modified Log-Payoff Function (MLPF), developing a precise valuation formula. The solution (20) provides a structured approach to solving complex differential equations in financial mathematics, particularly useful in option pricing and other related fields. Through

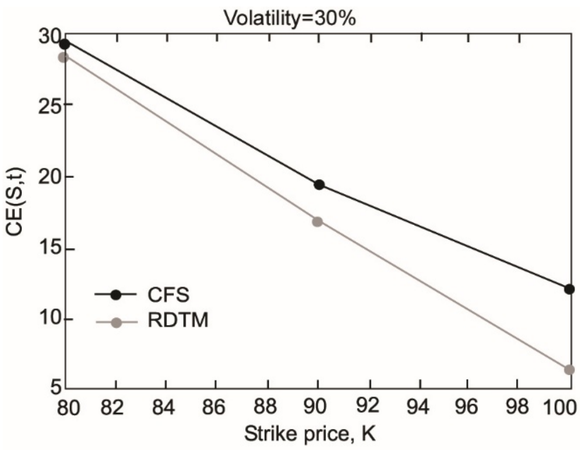

Fig. 1. Comparison of CFS and RDTM for volatility σ=30%.

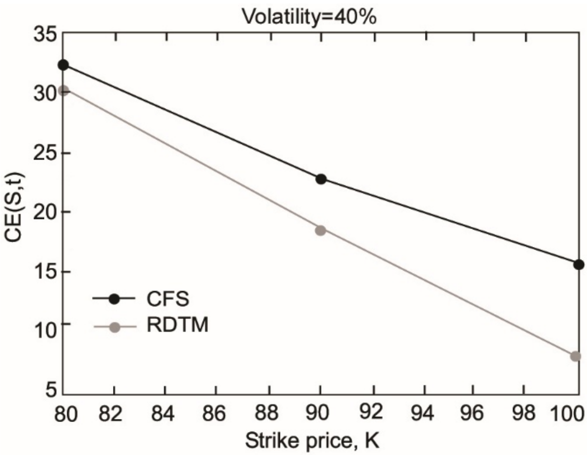

Fig. 2. Comparison of CFS and RDTM for volatility σ=40%.

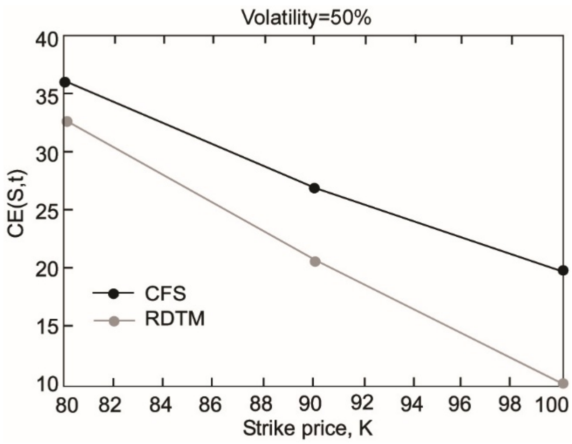

Fig. 3. Comparison of CFS and RDTM for volatility σ=50%.

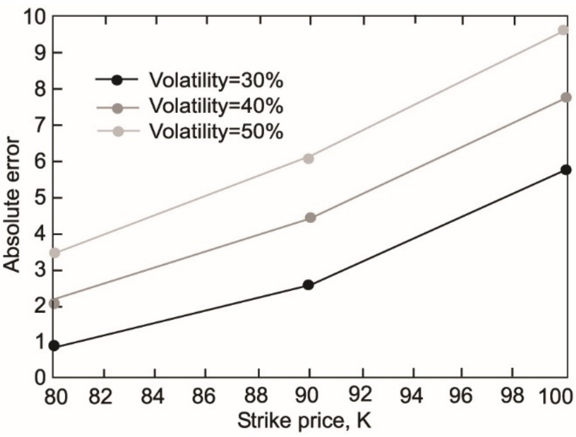

Fig. 4. Absolute error of RDTM for different values of volatility σ.

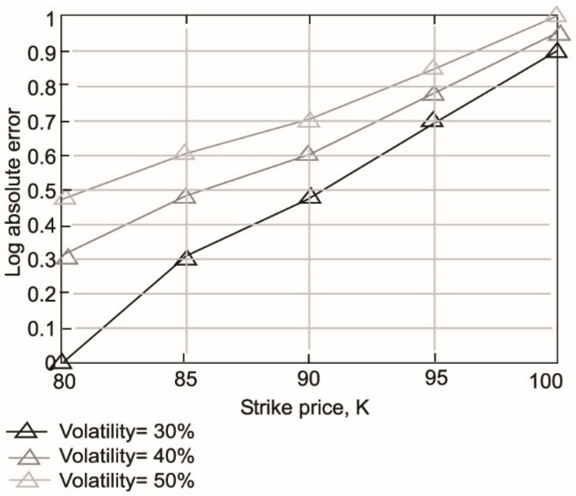

Fig. 5. The log plot of absolute error of RDTM for different values of volatility σ.

comprehensive comparisons illustrated in Figs. 1-3 between RDTM and the traditional Closed Form Solution (CFS). The analysis reveals strong agreement, particularly in scenarios of decreasing volatility. Notably, as volatility decreases and ESCOM moves deeper into profitability (in-the-money), significant reductions in absolute errors are observed across these figures. Conversely, Fig. 4 highlights that ESCOM prices escalate with higher volatility levels. Fig. 5 represents the log-absolute error, providing a clearer view of the error dynamics across different strike prices and volatility levels. This transformation helps in visualizing the relative differences more effectively, especially for smaller error values. These findings underscore RDTM's capability to efficiently and accurately price ESCOM with MLPF, affirming its role as a robust framework for delivering reliable financial solutions, both numerically and approximately.

Conflict of Interest. The authors declare that they have no conflict of interest.

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